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# Quasi-normal modes of warped black holes and warped AdS/CFT correspondence

**Bin Chen and Zhi-bo Xu**

*Department of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, P.R. China*

*E-mail:* bchen01@pku.edu.cn, xuzhibo@pku.edu.cn

ABSTRACT: We analytically calculate the quasi-normal modes of various perturbations of the space-like stretched and the null warped $AdS_3$ black holes. From AdS/CFT correspondence, these quasi-normal modes are expected to appear as the poles in momentum space of retarded Green's functions of dual operators in CFT at finite temperature. We find that this is indeed the case, after taking into account of the subtle identification of quantum numbers. The subtlety comes from the fact that only after appropriate coordinate transformation the asymptotic geometries of the warped black holes are the same as the ones of warped $AdS_3$. We show that in general the quasi-normal modes are in good agreement with the prediction of the warped AdS/CFT correspondence, up to a constant factor. As a byproduct, we compute the conformal dimensions of the boundary operators dual to the perturbations. Our results give strong support to the conjectured warped AdS/CFT correspondence.

KEYWORDS: AdS-CFT Correspondence, Black Holes

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## 1 Introduction

The study of the quasi-normal modes has been a classical subject in black hole physics [1]. They are called the "sound" of the black hole, characterizing the response of the black hole to various perturbations. As usual, the perturbations of the black holes obey linearized equations of motion. And the quasi-normal modes are defined as the perturbations subject to the physical boundary condition which states that near the horizon of the black hole the local solution is purely ingoing and at spatial infinity the solution is purely outgoing. As a result, the frequencies of the perturbations are complex, indicating that the perturbations undergo damped oscillations, just as the ring of a bell.

In fact, the frequencies of the quasi-normal modes usually take only a discrete set of complex values. The imaginary parts of the frequencies characterize the decay time of the perturbations. Or in other words, their inverses characterize the relaxation time of the system back to thermal equilibrium, with the black hole being taken as a thermal dynamic system. Especially for the black holes in Anti-de-Sitter(AdS) spacetime, the above picture has a nice realization in its dual field theory. From AdS/CFT correspondence [2], the black hole in the bulk corresponds to the quantum field theory on the boundary at a finite temperature. In recent years, the correspondence at finite temperature has been widely applied to the physical systems ranging from quark-gluon-plasma, superconductor, superfluid to cold atom physics [7]. The quasi-normal modes of the black holes correspond to the operators perturbing the thermal equilibrium in dual field theory [3]. In finite temperature field theory, the return to equilibrium under the small perturbations is described by linear response theory. The poles in the retarded green function of the perturbations in momentum space encode the information of relaxation process. From AdS/CFT correspondence, these poles are closely related to the quasi-normal frequencies of the black holes, as first suggested in [4]. The qualitative agreements via numerical analysis have been found for the black holes in AdS spacetime [4, 5]. Moreover in a remarkable paper [6], the quantitative agreement has been confirmed for the perturbations with various spins of the BTZ black hole.

Actually, the Banados-Teitelboim-Zanelli(BTZ) black hole sets up the first example of AdS/CFT correspondence. It is a solution of the vacuum Einstein equations in three-dimensional anti-de Sitter spacetime [8]. Its dual is a two-dimensional conformal field theory with independent left and right sectors. At thermal equilibrium, these two sectors may have different temperatures $(T_L, T_R)$. For a small perturbation by the operator $O$ with conformal weights $(h_L, h_R)$, its retarded Green's function has two sets of poles:

$$\begin{aligned} \omega_L &= k - 4\pi i T_L(n + h_L), \\ \omega_R &= -k - 4\pi i T_R(n + h_R), \end{aligned} \tag{1.1}$$

with $n$ being non-negative integer. In [6], it has been shown that these poles are in exact agreement with the quasi-normal frequencies of corresponding pertubations of the BTZ black hole.

Recently, inspired by the study of $AdS_3/CFT_2$ correspondence in the BTZ black hole, a new kind of warped $AdS_3/CFT_2$ correspondence has been proposed in [9]. It was pointed out that for the spacelike stretched and the null warped $AdS_3$ black holes, there exist dual two-dimensional conformal field theory descriptions. The proposal was supported by the study of thermodynamic on both sides. It was further conjectured that $\nu > 1$ quantum topological massive gravity is holographically dual to a two-dimensional conformal field theory with central charges $(c_L, c_R)$. However, this correspondence is intriguing in the sense that the warped $AdS_3$ spacetime has very different conformal boundary from the one of $AdS_3$. The naive expectation that the holographic CFT resides on the asymptotic boundary seems not true any more. The dictionary in warped AdS/CFT correspondence is not clear yet. As a step to understand the correspondence, we calculated the scalar quasi-normal modes of the spacelike stretched $AdS_3$ black holes in [10]. At the first looking, the quasi-normal modes we obtained are quite different from the prediction of the usual AdS/CFT correspondence. In this paper, we resolve this puzzle. The key point in our analysis is to notice that the asymptotic metric of the warped black holes is actually not the same as the one of the warped $AdS_3$ spacetime. One needs to make local coordinates transformations to identify two geometries. Such transformations induce the identifications of two sets of quantum numbers in two different backgrounds. After using the right quantum numbers, we find that the relations on the quasi-normal modes indeed are in good match with the prediction of (warped) AdS/CFT correspondence.

Moreover in this paper, we calculate other kinds of the quasi-normal modes of the warped black holes. Our calculation includes the vector and spinor quasi-normal modes of the spacelike stretched warped $AdS_3$ black holes, and also the scalar, vector and spinor quasi-normal modes of the null warped $AdS_3$ black holes. In all these cases, the quasi-normal modes could be obtained analytically. And after taking into account of the subtlety on the quantum numbers, all the quasi-normal modes are in good agreement with the prediction.

One subtle point in our study is on the boundary condition at the asymptotical infinity. In the usual AdS black hole case, the effective potential is infinitely high and one may impose vanishing Dirichlet condition at infinity on the eigenfunction. In the warped AdS black hole case, this is not obvious. The asymptotic boundary condition on the gravitational perturbation has been under intense study [19–21] to define the dual CFT. For other kinds of perturbations, one may just



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require the flux at asymptotic boundary to be finite. This turns out to be equivalent to the vanishing Dirichlet condition in some cases. But generically the finite flux condition leads to more quasi-normal modes.

To set up the correspondence, it is essential to have the conformal dimensions of corresponding operators. One way to obtain the conformal dimensions could be from the equations of motion of various perturbations and studying the asymptotic behavior of the solutions. In our case, there is another way to compute the conformal dimension. This way stems from the existence of the isometry algebra $SL(2,R)\times U(1)$ in the backgrounds, which allows us to identify the conformal dimensions of primary operators in the dual CFT.

The remaining part of the paper is organized as following: in the next section, we focus on the spacelike stretched warped black hole and analyze its quasi-normal modes. And in section 3, we turn to the study of the quasi-normal modes of the null warped black hole. In section 4, we compute the conformal dimensions of the operators dual to the perturbations from $SL(2,R)\times U(1)$ algebra. In section 5, we end with the conclusions and the discussions.

## 2 Quasi-normal modes of the spacelike stretched warped $AdS_3$ black hole

The spacelike stretched $AdS_3$ spacetime is the vacuum solution of three-dimensional topological massive gravity [11, 12]. This spacetime has an isometry group $SL(2)_R\times U(1)_L$. It could be a stable vacuum with appropriate boundary behavior, if the parameter $\nu>1$ [19]. Just as the BTZ black hole could be constructed as the orbifold of the $AdS_3$ spacetime, the black hole asymptotic to spacelike warped $AdS_3$ could be constructed from discrete identification as well. We would not like to review the construction here. The interested reader can find the details in [9].

The metric of the spacelike stretched warped $AdS_3$ black hole takes the following form in terms of Schwarzschild coordinates:

$$ds^2 = l^2(\mathrm{d}t^2 + 2M(r)\mathrm{d}t\mathrm{d}\theta + N(r)\mathrm{d}\theta^2 + D(r)\mathrm{d}r^2), \tag{2.1}$$

where

$$M(r) = \nu r - \frac{1}{2}\sqrt{r_+r_-(\nu^2+3)}, \tag{2.2}$$

$$N(r) = \frac{r}{4}\left(3(\nu^2-1)r + (\nu^2+3)(r_+ + r_-) - 4\nu\sqrt{r_+r_-(\nu^2+3)}\right), \tag{2.3}$$

$$D(r) = \frac{1}{(\nu^2+3)(r-r_+)(r-r_-)}, \tag{2.4}$$

and $-l^{-2}$ is a negative cosmological constant and the parameter $\nu = \mu l/3$ with $\mu$ being the mass of the graviton. Just like the BTZ black hole, there are two horizons located at $r = r_+$ and $r = r_-$. We will focus on the physical black holes without pathology, in which case we need to require $\nu > 1$. When $\nu = 1$, there is no stretching and the above black hole becomes the usual BTZ black hole. This kind of warped black hole was discussed in [14, 15], and its properties were studied in [16, 17]. The other recent studies could be found in [22].

In [9], the temperatures of the warped black holes were identified to be

$$\frac{1}{T_H} = \frac{4\pi\nu l}{\nu^2+3}\frac{T_L+T_R}{T_R}, \tag{2.5}$$



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where

$$T_L = \frac{(\nu^2+3)}{8\pi l}\left(r_+ + r_- - \frac{\sqrt{(\nu^2+3)r_+r_-}}{\nu}\right), \tag{2.6}$$

$$T_R = \frac{(\nu^2+3)(r_+ - r_-)}{8\pi l}, \tag{2.7}$$

are the temperature of the dual CFT. The dual two-dimensional CFT is supposed to have the central charges

$$c_L = \frac{l}{G}\frac{4\nu}{\nu^2+3}, \qquad c_R = \frac{l}{G}\frac{5\nu^2+3}{\nu(\nu^2+3)}. \tag{2.8}$$

It has been shown in [13, 20] that the above central charges could be obtained from central extended Virasoro algebra, based on the fact that the asymptotic symmetries of the geometries form a semi-product of a Virasoro algebra and a current algebra.

The scalar perturbation about this background obeys the equation of motion:

$$(\nabla_\mu \nabla^\mu - m^2)\Phi = 0. \tag{2.9}$$

Since the background has the translational isometry along $t$ and $\theta$, we may make the following ansatz

$$\Phi = e^{-i\omega t + ik\theta}\phi. \tag{2.10}$$

After introducing the variable

$$z = \frac{r - r_+}{r - r_-}, \tag{2.11}$$

we find that the equation of motion on $\phi$ is

$$z(1-z)\frac{d^2\phi}{dz^2} + (1-z)\frac{d\phi}{dz} + \frac{1}{(\nu^2+3)^2}\left(\frac{A}{z} + B + \frac{C}{1-z}\right)\phi = 0, \tag{2.12}$$

where

$$A = \frac{1}{(r_+ - r_-)^2}\left(2k + \omega\sqrt{r_+}\left(2\nu\sqrt{r_+} - \sqrt{\nu^2+3}\sqrt{r_-}\right)\right)^2, \tag{2.13}$$

$$B = -\frac{1}{(r_+ - r_-)^2}\left(2k + \omega\sqrt{r_-}\left(2\nu\sqrt{r_-} - \sqrt{\nu^2+3}\sqrt{r_+}\right)\right)^2, \tag{2.14}$$

$$C = 3(\nu^2-1)\omega^2 - m^2 l^2(\nu^2+3). \tag{2.15}$$

The solutions take the forms of hypergeometric function. Near the horizon, there are two independent solutions

$$\phi_1 = z^\alpha(1-z)^\beta F(a,b,c,z), \qquad \phi_2 = z^{-\alpha}(1-z)^\beta F(a-c+1, b-c+1, 2-c, z), \tag{2.16}$$

where

$$\alpha = -i\frac{\sqrt{A}}{\nu^2+3},$$

$$\beta = \frac{1}{2}\left(1 - \sqrt{1 - \frac{4C}{(\nu^2+3)^2}}\right),$$



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and

$$\begin{aligned} c &= 2\alpha + 1, \\ a &= \alpha + \beta + i\sqrt{-B}/(\nu^2 + 3), \\ b &= \alpha + \beta - i\sqrt{-B}/(\nu^2 + 3). \end{aligned}$$

The next step is to impose the physical boundary condition to determine the quasi-normal modes. From the definition, the quasi-normal modes have to be purely ingoing at the horizon. The eigenfunction $\phi_1$ satisfies this condition. On the other hand, the asymptotical boundary condition at $z = 1$ is not obvious. In [10], we imposed the requirement that the outgoing flux should be finite so that the coefficients of the divergent terms must vanish. This gives out two sets of quasi-normal modes, determined by the relation

$$c - a = -n, \qquad \text{or} \qquad c - b = -n, \tag{2.17}$$

with $n$ being a non-negative integer.

1) Case 1: $c - a = -n$
   In this case, we are led to the following equation on $\omega$:

$$-i\frac{1}{r_+ - r_-}\frac{1}{\nu^2+3}(4k + \omega\delta) + \frac{1}{2}\left(1 + \sqrt{1 - \frac{4C}{(\nu^2+3)^2}}\right) = -n, \tag{2.18}$$

   where

$$\delta \equiv 2\nu(r_+ + r_-) - 2\sqrt{(\nu^2+3)r_+ r_-}. \tag{2.19}$$

2) Case 2: $c - b = -n$
   In this case, the equation on $\omega$ is much simpler,

$$-n - \frac{1}{2} + i\frac{2\nu\omega}{\nu^2+3} = \frac{1}{2}\sqrt{1 - \frac{4C}{(\nu^2+3)^2}}, \tag{2.20}$$

   which has the solution

$$\omega_L = -i\left\{(2n+1)\nu + \sqrt{3\left(n + \frac{1}{2}\right)^2(\nu^2 - 1) + \left(\frac{1}{4} + \frac{m^2 l^2}{\nu^2+3}\right)(\nu^2+3)}\right\}. \tag{2.21}$$

   Note that the frequency is pure imaginary, being independent of the angular momentum.

If one tries to solve $\omega$ from the equation (2.18), one would get a solution of quite involved form. Obviously, the quasi-normal modes look very different from the prediction (1.1) of AdS/CFT correspondence.

However, this is just an illusion. We are going to show that the AdS/CFT correspondence still holds, but in a subtle way. Firstly let us consider the conformal weight of the scalar field of mass $m$, even though the general dictionary of warped AdS/CFT correspondence has not been



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set up. Consider the scalar field of mass $m$ propagating in the spacelike warped $AdS_3$, which has the metric

$$ds^2 = \frac{l^2}{\nu^2+3}\left[-(1+r^2)d\tau^2 + \frac{dr^2}{1+r^2} + \frac{4\nu^2}{\nu^2+3}(dx+rd\tau)^2\right]. \tag{2.22}$$

Near the boundary, with the ansatz

$$\Phi = e^{i(\tilde{k}x-\tilde{\omega}\tau)}\phi, \tag{2.23}$$

the scalar equation takes the form

$$\partial_z^2\phi + \left(\tilde{\omega}^2 - \frac{2\tilde{k}\tilde{\omega}}{z} - \frac{s_s}{z^2}\right)\phi = 0, \tag{2.24}$$

where

$$s_s = \frac{3(1-\nu^2)}{4\nu^2}\tilde{k}^2 + \frac{l^2}{\nu^2+3}m^2. \tag{2.25}$$

At the asymptotical region, $\phi \sim r^{\Delta_s}$. To have a well-behaved solution, $\Delta_s$ should be negative. In the end, we have the relation

$$h_R = -\Delta_s = \frac{1}{2} \pm \sqrt{\frac{1}{4} + s_s}. \tag{2.26}$$

This is the conformal weight of the scalar field of mass $m$. We will present another derivation of the conformal dimension of the primary operator dual to the scalar perturbation in section 4.

The subtlety comes from the asymptotic behavior of the spacelike stretched warped $AdS_3$ black hole. The metric of the asymptotic geometry of the black hole is of the form

$$\frac{ds^2}{l^2} = \frac{3(\nu^2-1)r^2d\theta^2}{4} + \frac{dr^2}{(\nu^2+3)r^2} + dt^2 + 2\nu r dt d\theta. \tag{2.27}$$

It looks different from the spacelike warped $AdS_3$. However, after proper identification, it is actually the same as the asymptotic geometry of the spacelike warped $AdS_3$. Locally, the identification is [19]

$$\tau \leftrightarrow -\frac{\nu^2+3}{2}\theta, \qquad x \leftrightarrow -\frac{\nu^2+3}{2\nu}t. \tag{2.28}$$

This identification suggests that in the warped black hole case, the correspondence relation should be modified. Either from the scalar equation in (2.27), or from the above identification, we have the following relations between the quantum numbers in two backgrounds:

$$\tilde{\omega} = \frac{2}{\nu^2+3}k, \qquad \tilde{k} = \frac{2\nu}{\nu^2+3}\omega. \tag{2.29}$$

Notice that the identification (2.28) between the asymptotic geometries is local. In fact, as pointed out in [19] the global warped $AdS_3$ is not the ground state of the warped black hole. In other words, the spacelike stretched $AdS_3$ black holes with mass and angular momentum could not be taken as the "excited" states. This is reflected in the fact that the Killing vectors $\partial_\tau$ and $\partial_x$ of the warped $AdS_3$ spacetime are translations along the noncompact orbits, while the Killing vector $\partial_\theta$ of the warped black hole is along the compact orbit. Consequently, the quantum number $\tilde{\omega}$ is continuous, while the quantum number $k$ should be integer-valued. However, the identification (2.29)



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relating these two quantities stems from the local identification and does not care about the global properties. We will see shortly that it is the identification (2.29) that make the conjectured warped AdS/CFT correspondence manifest in the context of the quasi-normal modes.

The relations (2.29) allows us to reorganize (2.18) into

$$\tilde{\omega}_R = \frac{1}{\nu^2+3}(-4\pi T_L l\tilde{k} - (i4\pi T_R l)(n+h_R)). \tag{2.30}$$

This is not exactly, but quite similar to (1.1). The discrepancy is the $1/(\nu^2+3)$ factor, which may be from the warped geometry or the coordinates we choose which may induce the redefinition of the temperature. Anyway, we would like to take (2.30) as the convincing evidence to support the warped AdS/CFT correspondence.

Moreover, we have another set of the quasi-normal modes determined by (2.20). However, in this case, due to the absence of the quantum number $k$, there is no relation on $\tilde{\omega}_L$. In fact, the relation (2.20) gives

$$\tilde{k} = -i(n+h_L), \tag{2.31}$$

where $h_L = h_R$ for scalar. The fact that there is only one set of the quasi-normal modes sounds strange. This could be related to the fact that the isometry group of the spacelike warped $AdS_3$ is $\mathrm{U}(1)_L \times \mathrm{SL}(2)_R$.

We would like to clarify the discrepancy between the relations (2.30), (2.31) and the relation (1.1) furthermore. The strangest thing is the appearance of left-moving temperature $T_L$ in (2.30) rather than in (2.31). This is mainly due to the special property of the dual 2D CFT. In fact, the existence of nonvanishing angular momentum in the warped black holes induce a chemical potential in the right-moving sector. The scalar operator in 2D CFT not only has conformal weights, but also has the right charge coupled to the chemical potential. More precisely the chemical potential $\Omega_R = -2\pi T_L, q_R = \tilde{k}$. While the temperature $T_L$ in the relation (2.31) could be recovered by define the left-moving frequency in 2D CFT as $\omega_L = 2\pi T_L \tilde{k}$. This picture is inspired by the study in Kerr/CFT correspondence [26] and will be discussed more clearly in our future work [27].

In the special case of zero mass black hole, the left-moving temperature $T_L \propto M^{ADT} = 0$, the dual CFT becomes a "chiral" one, similar to the one dual to the null warped background. Now the chemical potential is zero, and the relation (2.30) is in better match with (1.1). However, unlike the null warped black holes we will study in the next section, there still exist a left sector with (2.31).

Another interesting point is that one could also choose the conformal weight to be

$$h_R = \frac{1}{2} - \sqrt{\frac{1}{4} + s_s}, \tag{2.32}$$

if $-\frac{1}{4} < s_s < 0$. In this case, the finiteness of the flux requires that

$$a = -n, \quad \text{or} \qquad b = -n. \tag{2.33}$$

Similarly, we get the above two relations (2.30), (2.31). It is remarkable that one cannot get this conclusion from imposing the vanishing Dirichlet condition at asymptotic infinity. This fact suggests that the requirement of finite flux is not only physical but also more powerful.

Before ending the discussion on the scalar quasi-normal modes, we would like to elucidate the organization of the quasi-normal relations (2.18), (2.20) in terms of $\tilde{\omega}$ and $\tilde{k}$. The essential point

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is that the warped AdS/CFT correspondence states that quantum gravity asymptotic to the warped $AdS_3$ spacetime is holographically dual to 2D CFT. Therefore, in setting up the dictionary, one needs to use the quantum numbers of the warped $AdS_3$. More technically, the quantum number $\tilde{\omega}$ and $\tilde{k}$ of bulk warped $AdS_3$ spacetime correspond to the eigenvalues of $\bar{L}_0$ and $L_0$ in dual field theory. Once we have the black hole in the bulk and change the local geometry, we should still use the quantum numbers $\tilde{\omega}$ and $\tilde{k}$ in the study of warped AdS/CFT correspondence at finite temperature.

In the remaining part of this section, we calculate the vector and fermionic quasi-normal modes of the spacelike $AdS_3$ black holes and rewrite them in terms of $\tilde{\omega}$ and $\tilde{k}$.

### 2.1 Vector perturbation

In order to obtain the quasi-normal modes of the vector fields, we should study the equations of the massive vector fields which are second order ordinal differential equations

$$\nabla_\mu F^{\mu\nu} = m^2 A^\nu. \tag{2.34}$$

However, one may work with the following first order equations whose solutions are the solutions of the above equations in three-dimensional spacetime:

$$\epsilon_\lambda^{\ \alpha\beta}\partial_\alpha A_\beta = -mA_\lambda, \tag{2.35}$$

where $\epsilon_\lambda^{\ \alpha\beta}$ is the Levi-Civita tensor with $\epsilon^{tr\theta} = 1/\sqrt{-g}$. Since the background have translational symmetries along $t$ and $\theta$, we can make the following ansatz

$$A_\mu = e^{-i\omega t + ik\theta}\phi_\mu. \tag{2.36}$$

Then the equations of motion can be given explicitly

$$\begin{aligned}
\frac{\mathrm{d}\phi_t}{\mathrm{d}r} &= 2D(r)\left(\left(\frac{-\omega k}{ml} + mlM(r)\right)\phi_t - \left(\frac{\omega^2}{ml} + ml\right)\phi_\theta\right), &(2.37)\\
\frac{\mathrm{d}\phi_\theta}{\mathrm{d}r} &= 2D(r)\left(\left(\frac{k^2}{ml} + mlN(r)\right)\phi_t - \left(\frac{-\omega k}{ml} + mlM(r)\right)\phi_\theta\right), &(2.38)\\
\phi_r &= -\frac{2D(r)}{ml}(ik\phi_t + i\omega\phi_\theta). &(2.39)
\end{aligned}$$

After changing the variables to

$$z = \frac{r - r_+}{r - r_-}, \tag{2.40}$$

we have the second order ordinary equation for $\phi_t$

$$z(1-z)\frac{d^2\phi_t}{dz^2} + (1-z)\frac{d\phi_t}{dz} + \left(\frac{A_v}{z} + B_v + \frac{C_v}{1-z}\right)\phi_t = 0, \tag{2.41}$$

where

$$\begin{aligned}
A_v &= \frac{1}{(r_+ - r_-)^2(\nu^2+3)^2}\left(2k + \omega\sqrt{r_+}(2\nu\sqrt{r_+} - \sqrt{\nu^2+3}\sqrt{r_-})\right)^2,\\
B_v &= -\frac{1}{(r_+ - r_-)^2(\nu^2+3)^2}\left(2k + \omega\sqrt{r_-}(2\nu\sqrt{r_-} - \sqrt{\nu^2+3}\sqrt{r_+})\right)^2,\\
C_v &= \frac{1}{(\nu^2+3)^2}\left(3(\nu^2-1)\omega^2 - (m^2l^2 + 2m\nu l)(\nu^2+3)\right). &(2.42)
\end{aligned}$$



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The solutions can be written in terms of hypergeometric functions. There are two independent solutions,

$$\phi_1 = z^{\alpha_v}(1-z)^{\beta_v+1}F(a_v+1, b_v+1, c_v, z), \qquad \phi_2 = z^{-\alpha_v}(1-z)^{\beta_v+1}F(a_v-c_v+2, b_v-c_v+2, 2-c_v, z), \tag{2.43}$$

where $\alpha_v = -i\sqrt{A_v}$, $\beta_v = (-1+\sqrt{1-4C_v})/2$ and

$$c_v = 1+2\alpha_v, \quad a_v = \alpha_v+\beta_v+i\sqrt{-B_v}, \quad b_v = \alpha_v+\beta_v-i\sqrt{-B_v}. \tag{2.44}$$

Since the quasi-normal modes are purely ingoing at the horizon, $\phi_1$ is the solution we need.

By using the equation (2.37), we obtain $\phi_\theta$ in terms of the variable $z$ ,

$$\phi_\theta = \tilde{A}_v\phi_t + \tilde{B}_v\frac{1}{1-z}\phi_t + \tilde{C}_v z\frac{\mathrm{d}\phi_t}{\mathrm{d}z}, \tag{2.45}$$

where

$$\begin{aligned}
\tilde{A}_v &= \frac{1}{2\omega^2+2m^2l^2}\left(-2\omega k + 2m^2l^2 v r_- - m^2l^2\sqrt{r_+r_-(v^2+3)}\right),\\
\tilde{B}_v &= \frac{m^2l^2v(r_+-r_-)}{\omega^2+m^2l^2},\\
\tilde{C}_v &= -\frac{ml(v^2+3)(r_+-r_-)}{2\omega^2+2m^2l^2}.
\end{aligned}$$

Similarly the solution can be written in terms of hypergeometric functions explicitly. Finally we have

$$\begin{aligned}
\phi_\theta &= z^{\alpha_v}(1-z)^{\beta_v}\Big\{(\tilde{A}_v+\tilde{C}_v(\alpha_v+\beta_v-b_v))(c_v-b_v-1)F(a_v,b_v,c_v,z)\\
&\quad+\left(2\beta_v(\tilde{A}_v+\tilde{C}_v(\alpha_v+\beta_v-b_v))+a_v\tilde{C}_v(c_v-a_v-1)\right)F(a_v,b_v+1,c_v,z)\\
&\quad+a_v\frac{ml(r_+-r_-)}{2\omega^2+2m^2l^2}\left(2mlv-(v^2+3)\beta_v\right)F(a_v+1,b_v+1,c_v,z)\Big\} \qquad (2.46)\\
\phi_t &= a_v z^{\alpha_v}(1-z)^{\beta_v+1}F(a_v+1,b_v+1,c_v,z). \qquad (2.47)
\end{aligned}$$

There are two special cases we would like to consider separately. If $a_v = 0$ or $c_v - b_v - 1 = 0$ which may lead to $\omega^2 + m^2l^2 = 0$, one can directly solve the equations. Here we introduce a new parameter $\tilde{\beta}_v = \frac{2mlv}{v^2+3}$ for convenience. In the case $\omega = -iml$, we have $\tilde{\beta}_v = \beta_v$ which means $a_v = 0$. In this case, one of the solutions of the equations of motion is

$$\phi_t = 0, \quad \phi_\theta = z^{\alpha_v}(1-z)^{\tilde{\beta}_v}. \tag{2.48}$$

This solution has purely ingoing mode at the horizon and so is the right solution we want. While the other solution is that $\phi_t = z^{-\alpha_v}(1-z)^{-2mlv}$ and $\phi_\theta$ can be obtained from the equations of motion correspondingly. One can find that $\phi_\theta$ approaches to $z^{-\alpha_v}$ near the horizon and $(1-z)^{-1-2mlv}$ at infinity. This is a solution with outgoing mode near the horizon. Thus it is not the solution we need.

In the case $\omega = iml$, one has $c_v - b_v - 1 = 0$. Then one solution is $\phi_t = 0$, $\phi_\theta = z^{-\alpha_v}(1-z)^{\beta_v}$. It is a solution with outgoing mode. The other solution is a solution with ingoing mode, where $\phi_t = z^{\alpha_v}(1-z)^{-2mlv}$ and $\phi_\theta$ approaches to $(1-z)^{-1-2mlv}$ at asymptotic infinity.

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One has to impose the physical boundary condition at asymptotic infinity. One may require the flux vanishing condition as the scalar field case. Let us consider the energy flux and the angular momentum flux. They are defined as

$$\mathcal{F}_e = \int dt d\theta \sqrt{-g} T^r_t, \quad \mathcal{F}_a = \int dt d\theta \sqrt{-g} T^r_\theta. \tag{2.49}$$

Using the equations of motion and considering the flux at infinity after time averaging, we have

$$\mathcal{F}_e \simeq (ik\phi_t + i\omega\phi_\theta)\phi_t^* + c.c., \tag{2.50}$$

$$\mathcal{F}_a \simeq (ik\phi_t + i\omega\phi_\theta)\phi_\theta^* + c.c\ldots \tag{2.51}$$

For $\omega$ is real, the leading term of the flux at asymptotic infinity is proportional to

$$\left|\frac{\Gamma(c_\nu)\Gamma(a_\nu + b_\nu + 2 - c_\nu)}{\Gamma(a_\nu + 1)\Gamma(b_\nu + 1)}\right|^2 (1-z)^{-1-2\beta_\nu} \tag{2.52}$$

Generally the finite flux boundary condition for the solutions at asymptotic infinity gives the following relation:

$$b_\nu + 1 = -n, \quad \text{or} \qquad a_\nu = -n. \tag{2.53}$$

Note that $a_\nu = 0$ also satisfies the boundary condition from the solution (2.48).

For $\nu = 1$, we need some special considerations since the third line of (2.46) may be zero. In the end we find that it leads to the same quasi-normal modes as the ones in the BTZ black hole, once the proper redefinition of the temperature is taken into account.

1. For the case $b_\nu + 1 = -n$, we have

$$-i\frac{1}{r_+ - r_-}\frac{1}{\nu^2+3}(4k + \omega\delta) + \frac{1}{2}\left(1 + \sqrt{1 - 4C_\nu}\right) = -n, \tag{2.54}$$

which is very similar to (2.18) except replacing $C$ with $C_\nu(\nu^2+3)^2$.

2. For the case $a_\nu = -n$, we have

$$i\frac{2\nu\omega}{\nu^2+3} = n - \frac{1}{2} + \frac{1}{2}\sqrt{1 - 4C_\nu}, \tag{2.55}$$

For the general case $\nu > 1$, the relations (2.54), (2.55) lead to quite involved forms of the frequencies of the quasi-normal modes. However, as we stated above, we have to consider the subtle identification of the quantum numbers. As the first step to compare the above results with the CFT prediction, we would like to discuss the conformal weights of the massive vectors. From the massive vector field equation in the spacelike stretched $AdS_3$ spacetime, after analyzing the behavior of the solution at asymptotic infinity, we can determine its conformal weight. One choice is

$$h_R^\nu = \frac{1}{2} + \sqrt{\frac{1}{4} + s_\nu} \tag{2.56}$$

with

$$s_\nu = \frac{3(1-\nu^2)}{4\nu^2}\tilde{k}^2 + \frac{(m^2l^2 + 2\nu ml)}{\nu^2+3}. \tag{2.57}$$



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Therefore, taken into account of the subtle identification (2.29), the above relation (2.53) could be rewritten as

$$\tilde{\omega}_R^v = \frac{1}{v^2+3}(-4\pi T_L l\tilde{k} - (i4\pi T_R l)(n + h_R^v)), \quad \text{or} \tag{2.58}$$

$$\tilde{k} = -i(n + h_L^v). \tag{2.59}$$

where $h_L^v = h_R^v - 1$.

As the scalar case, there is another choice of the conformal weight

$$h_R^v = \frac{1}{2} + \sqrt{\frac{1}{4} + s_v'}, \tag{2.60}$$

where

$$s_v' = \frac{3(1-v^2)}{4v^2}\tilde{k}^2 + \frac{(m^2l^2 - 2vml)}{v^2+3}, \tag{2.61}$$

which corresponds to the vector field with a different helicity. In this case, the vanishing boundary condition cannot give the right constraint. One has to read from flux finiteness condition. In the end, one obtains the same relations (2.58) with $h_L^v = h_R^v + 1$.

### 2.2 Fermion perturbation

In this subsection we analyze the quasi-normal modes of the fermionic fields on the spacelike stretched warped $AdS_3$ black holes background. In order to solve the Dirac equations, we should choose the vielbein for the background spacetime and calculate the corresponding spin connection.

The vielbein $e_\mu^a$ is chosen as

$$e^0 = \frac{l}{2\sqrt{D(r)}}d\theta, \quad e^1 = l\sqrt{D(r)}dr, \quad e^2 = ldt + M(r)ld\theta, \tag{2.62}$$

where $e_\mu^a dx^\mu = e^a$. The spin connection can be calculated straightforwardly. The nonvanishing components of the spin connection are

$$\omega_t^{01} = -\omega_t^{10} = -M', \qquad \omega_r^{02} = -\omega_r^{20} = -\sqrt{D}M',$$
$$\omega_\theta^{01} = -\omega_\theta^{10} = MM' - N', \quad \omega_\theta^{12} = -\omega_\theta^{21} = \frac{-M'}{2\sqrt{D}}.$$

The Dirac equations are

$$\gamma^a e_a^\mu\left(\partial_\mu + \frac{1}{2}\omega_\mu^{ab}\Sigma_{ab}\right)\Psi + m\Psi = 0, \tag{2.63}$$

where $\Sigma_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$, $\gamma^0 = i\sigma^2$, $\gamma^1 = \sigma^1$, $\gamma^2 = \sigma^3$. Similarly, we change to the variable

$$z = \frac{r - r_+}{r - r_-},$$

and with the ansatz $\Psi = (\psi_+, \psi_-)e^{-i\omega t + ik\theta}$, we then have

$$\frac{d\psi_\pm}{dz} = \left(\pm\frac{2i(\omega v r_+ - \omega e + k)}{z(v^2+3)(r_+ - r_-)} \pm \frac{2i\omega v}{(v^2+3)(1-z)} - \frac{1}{2(1-z)} - \frac{1}{4z}\right)\psi_\pm$$
$$+\left(\mp i\omega + ml - \frac{v}{2}\right)\frac{1}{\sqrt{(v^2+3)z}(1-z)}\psi_\mp, \tag{2.64}$$



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where $e = \frac{1}{2}\sqrt{r_+ r_-(v^2+3)}$.

The solutions of these equations with only ingoing flux at the horizon are given by hypergeometric functions

$$\psi_+ = z^{\alpha_f+1/2}(1-z)^{\beta_f}F(a_f+1, b_f, c_f+1, z), \tag{2.65}$$

$$\psi_- = \frac{c_f(i\omega+ml-v/2)}{a_f(b_f-c_f)\sqrt{v^2+3}}z^{\alpha_f}(1-z)^{\beta_f}F(a_f, b_f, c_f, z), \tag{2.66}$$

where $c_f = 2\alpha_f + 1,\ a_f = \alpha_f + \beta_f + \gamma_f,\ b_f = \alpha_f + \beta_f - \gamma_f$, and

$$\alpha_f = -\frac{2i(\omega v r_+ - \omega e + k)}{(v^2+3)(r_+-r_-)} - \frac{1}{4}, \tag{2.67}$$

$$\beta_f = \frac{1}{2} - \sqrt{\frac{(ml-v/2)^2}{v^2+3} - \frac{3\omega^2(v^2-1)}{(v^2+3)^2}}, \tag{2.68}$$

$$\gamma_f = \frac{2i(\omega v r_- - \omega e + k)}{(v^2+3)(r_+-r_-)} - \frac{1}{4}. \tag{2.69}$$

Similar to the vector case, there are two special cases: $c_f - b_f = 0$ and $a_f = 0$, both of which lead to $\omega^2 + (ml - \frac{1}{2})^2 = 0$. In fact, if $Im\omega \leq 0$ then one has $c_f - b_f = 0$, while we have $a_f = 0$ for $Im\omega \geq 0$.

For the case $i\omega = ml - \frac{1}{2}$, one has a solution with purely ingoing mode near the horizon as $\psi_+ = 0,\ \psi_- = z^{\alpha_f}(1-z)^{\tilde{\beta}_f}$, where $\tilde{\beta}_f = \frac{(2ml-v)v}{v^2+3} + \frac{1}{2}$. The other solution has outgoing mode near the horizon.

For the case $-i\omega = ml - \frac{1}{2}$, one solution with outgoing mode is $\psi_- = 0,\ \psi_+ = z^{-\alpha_f-\frac{1}{2}}(1-z)^{\tilde{\beta}_f}$. The other one which has ingoing mode is $\psi_+ = z^{\alpha_f}(1-z)^{1-\tilde{\beta}_f}$ and $\psi_-$ has the same asymptotic behavior of $\psi_+$ near the horizon and at infinity.

We impose the vanishing flux condition at asymptotic infinity. The flux is

$$\sqrt{-g}\bar{\Psi}e_1^r\gamma^1\Psi \cong (1-z)^{-1}(|\psi_+|^2 - |\psi_-|^2). \tag{2.70}$$

The leading divergent term of the flux is of order $(1-z)^{2\beta_f-1}$, where if $\beta_f$ is complex we choose $Re(\beta_f) < 1/2$ for the branch cut. Its coefficient

$$\left|\frac{\Gamma(c_f+1)\Gamma(c_f-a_f-b_f)}{\Gamma(c_f-a_f)\Gamma(c_f-b_f+1)}\right|^2 \tag{2.71}$$

must vanish. Considering that the solution in the $c_f - b_f = 0$ case also satisfy the boundary condition, so we have

$$c_f - a_f = -n, \text{ or } c_f - b_f = -n. \tag{2.72}$$

for the quasi-normal modes.

1. In the case: $c_f - b_f = -n$, we have

$$-n + i\frac{2v\omega}{v^2+3} = \sqrt{\frac{(ml-v/2)^2}{v^2+3} - \frac{3\omega^2(v^2-1)}{(v^2+3)^2}}, \tag{2.73}$$



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2. In the case: $c_f - a_f = -n$, we have

$$-i\frac{l}{r_+ - r_-}\frac{1}{\nu^2+3}(4k+\omega\delta) + \frac{1}{2} + \sqrt{\frac{(ml-\nu/2)^2}{\nu^2+3} - \frac{3\omega^2(\nu^2-1)}{(\nu^2+3)^2}} = -n. \tag{2.74}$$

In the limit $\nu = 1$, the spectrum of the quasi-normal mode is

$$\omega_R = -\frac{4k}{\delta} - i\frac{4(r_+ - r_-)}{\delta}\left(n + \frac{1}{2}(1 + |ml - 1/2|)\right), \tag{2.75}$$

$$\omega_L = -2i\left(n + \frac{1}{2}|ml - 1/2|\right) \tag{2.76}$$

where $\delta = 2(\sqrt{r_+} - \sqrt{r_-})^2$. So the left and the right conformal weights are given by $h_L = |ml - 1/2|/2$, and $h_R = (1 + |ml - 1/2|)/2$. This is in precise match with the results in the BTZ black hole obtained in [6], after considering the different choice of the vielbeins.

For the general case with $\nu > 1$, in order to compare with the prediction of warped AdS/CFT correspondence, we need the conformal weights of the massive fermionic operators. In the similar spirit as the scalar and the vector, we have

$$h_R^f = \frac{1}{2} + \sqrt{\frac{(ml-\nu/2)^2}{\nu^2+3} - \frac{3\tilde{k}^2(\nu^2-1)}{(\nu^2+3)^2}}. \tag{2.77}$$

Taken into account of the identification (2.29), the relation (2.74) is of the form

$$\tilde{\omega}_R^f = \frac{1}{\nu^2+3}\left(-4\pi T_L l\tilde{k} - (i4\pi T_R l)(n + h_R^f)\right), \tag{2.78}$$

and the relation (2.73) is of the form

$$\tilde{k}^f = -i(n + h_L^f), \tag{2.79}$$

where $h_L^f = h_R^f - \frac{1}{2}$.

Similarly, one may have

$$h_R^f = \frac{1}{2} + \sqrt{\frac{(ml+\nu/2)^2}{\nu^2+3} - \frac{3\tilde{k}^2(\nu^2-1)}{(\nu^2+3)^2}}, \tag{2.80}$$

and $h_L^f = h_R^f - \frac{1}{2}$. Even in this case, the quasi-normal modes are still given by (2.78), (2.79).

Let us summarize the result we obtained in this section. No matter what kind of the perturbations we considered, the quasi-normal modes of the spacelike stretched $AdS_3$ black hole could be simply written as

$$\tilde{\omega}_R = \frac{1}{\nu^2+3}(-4\pi T_L l\tilde{k} - (i4\pi T_R l)(n_1 + h_R)), \tag{2.81}$$

$$\tilde{k} = -i(n_2 + h_L), \tag{2.82}$$

with $n_1, n_2$ being non-negative integers.

## 3 Quasi-normal modes of the null warped black holes

Null warped $AdS_3$ spacetime is another vacuum solution of three-dimensional topological massive gravity.[1] It is only well defined at $\nu = 1$. Similar to other warped $AdS_3$ spacetime, it also has

[1] For the related study on AdS wave solution, see [18].



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isometry group $SL(2,R)\times U(1)_{\text{null}}$. The null warped black hole could be taken as the quotient of the null warped $AdS_3$. The metric of the null warped black hole is of the form

$$\frac{ds^2}{l^2} = -2r\mathrm{d}\theta\mathrm{d}t + (r^2+r+\alpha^2)\mathrm{d}\theta^2 + \frac{\mathrm{d}r^2}{4r^2}, \tag{3.1}$$

where $1/2 > \alpha > 0$ in order to avoid the naked causal singularity. The horizon is located at $r = 0$. From the thermodynamics of this black hole, it was argued that there exist non-vanishing right-moving temperature

$$T_R = \frac{\alpha}{\pi l}. \tag{3.2}$$

One may propose the following conjecture: $\nu = 1$ quantum topological massive gravity with asymptotical null warped $AdS_3$ geometry is holographically dual to a 2D boundary CFT with the left-moving central charge $c_L = \frac{l}{G}\frac{4\nu}{\nu^2+3}$ and the right-moving central charge $c_R = \frac{(5\nu^2+3)l}{G\nu(\nu^2+3)}$. From the black hole entropy, it seems that it is not necessary to have left-moving central charge since $T_L = 0$. However, the diffeomorphism anomaly requiring that $c_L - c_R = -\frac{l}{G\nu}$ asks for the existence of the left-moving sector.

In order to check this conjectured correspondence, we study the quasi-normal modes in the null warped black hole in this section. Firstly let us consider the scalar perturbation. The equation of motion for the scalar field is

$$\nabla^2\Phi - m^2\Phi = 0. \tag{3.3}$$

Taken the ansatz $\Phi = e^{-i\omega t+ik\theta}R(r)$, the equation becomes

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(4r^2\frac{\mathrm{d}}{\mathrm{d}r}R\right) + \left(-\frac{2\omega k}{r} + \frac{\omega^2(r^2+r+\alpha^2)}{r^2} - m^2l^2\right)R = 0. \tag{3.4}$$

The above equation can be solved by Kummer functions

$$R_\pm = e^{-\frac{z}{2}}z^{\frac{1}{2}\pm\tilde{m}_s}F\left(\frac{1}{2}\pm\tilde{m}_s-\kappa, 1\pm 2\tilde{m}_s, z\right) \tag{3.5}$$

where $z = -i\omega\alpha\frac{1}{r}$, $\tilde{m}_s = \frac{1}{2}\sqrt{1+m^2l^2-\omega^2}$ and $\kappa = \frac{i}{4\alpha}(\omega-2k)$. Here we choose $-\pi < argz < \pi$ for the branch cut. Actually the solution should be a combination

$$R = C_1R_+ + C_2R_-. \tag{3.6}$$

Now let us consider the boundary condition for the quasi-normal modes. There have to be only ingoing modes near the horizon where $z$ approaches to the infinity. The asymptotic expansion of Kummer function at asymptotic infinity is

$$F(\alpha,\gamma,z) \sim \frac{\Gamma(\gamma)}{\Gamma(\gamma-\alpha)}e^{-i\alpha\pi}z^{-\alpha} + \frac{\Gamma(\gamma)}{\Gamma(\alpha)}e^zz^{\alpha-\gamma}. \tag{3.7}$$

where it requires $-\frac{3\pi}{2} < argz < \frac{\pi}{2}$. In our case, the second term of the right hand equation corresponds to the outgoing modes near the horizon. For the solution (3.6), the vanishing outgoing condition requires

$$C_1 = -\frac{\Gamma(1-2\tilde{m}_s)}{\Gamma(\frac{1}{2}-\tilde{m}_s-\kappa)}C, \quad C_2 = \frac{\Gamma(1+2\tilde{m}_s)}{\Gamma(\frac{1}{2}+\tilde{m}_s-\kappa)}C, \tag{3.8}$$



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where $C$ is a constant. Next we require the flux at infinity to be vanishing. The flux is given by

$$\mathcal{F} \sim \frac{2\pi}{i} r^2 (\Phi^* \partial_r \Phi - c.c.) \tag{3.9}$$

For the solution $R(z)$, the leading term of the corresponding flux is proportional to $C_1^* C_2 - c.c.$ and the sub-leading term is proportional to $C_2 C_2^* r^{-1+2\tilde{m}_s}$. So if $Re(\tilde{m}_s) > \frac{1}{2}$, the flux vanishes if $C_2 = 0$ that is

$$\frac{1}{2} + \tilde{m}_s - \kappa = -n \tag{3.10}$$

We will see that $\frac{1}{2} + \tilde{m}_s$ is the conformal weight of the scalar of mass $m$ in the following and next section. While for the case $Re(\tilde{m}_s) < \frac{1}{2}$, the flux vanishes when $C_1 = 0$ or $C_2 = 0$, which indicate that

$$\frac{1}{2} + \tilde{m}_s - \kappa = -n, \text{or } \frac{1}{2} - \tilde{m}_s - \kappa = -n. \tag{3.11}$$

Note that for the case $Re(\tilde{m}_s) < \frac{1}{2}$, there are two possible choices of the conformal weights after considering the identification of the quantum numbers: $h_R = \frac{1}{2} \pm \tilde{m}_s$. Therefore, the above relations on quasi-normal frequencies could be simply written into

$$h_R - \kappa = -n. \tag{3.12}$$

In order to compare the quasi-normal mods with the result from the dual conformal field theory, we first need to identify the conformal weights of the dual operators. Now we calculate the conformal weight of the scalar field of mass $m$ from its asymptotic behaviors. The metric of the null warped $AdS_3$ spacetime could be of the following form

$$\frac{ds^2}{l^2} = \frac{du^2}{u^2} + \frac{dx^+ dx^-}{u^2} + \left(\frac{dx^-}{u^2}\right)^2. \tag{3.13}$$

Near the asymptotic region, we have the scalar equation of motion:

$$4r^2 \frac{d^2}{dr^2} R + 8r \frac{d}{dr} R + (4\tilde{k}_n^2 - m^2 l^2) R = 0, \tag{3.14}$$

where we have introduced $u^2 = 1/r$ and have made the following ansatz:

$$\Phi = e^{-i\tilde{\omega}_n x^- + i\tilde{k}_n x^+} R(r). \tag{3.15}$$

The subscript $n$ is introduced to denote the null background. The eigenfunction is of the form $r^{\Delta_s}$ with $-\Delta_s$ being the conformal weight. The equation (3.14) leads to

$$\Delta_s^2 + \Delta_s + \frac{4\tilde{k}_n^2 - m^2 l^2}{4} = 0, \tag{3.16}$$

with the solution

$$\Delta_s = -\frac{1}{2} \pm \frac{1}{2}\sqrt{1 + m^2 l^2 - 4\tilde{k}_n^2}. \tag{3.17}$$

In order that the solution is well-behaved, $\Delta$ should be negative, this helps us to take

$$\begin{aligned} \Delta_s &= -\frac{1}{2} \pm \frac{1}{2}\sqrt{1 + m^2 l^2 - 4\tilde{k}_n^2}, \qquad \text{if } m^2 < 4\tilde{k}_n^2, \\ \Delta_s &= -\frac{1}{2} - \frac{1}{2}\sqrt{1 + m^2 l^2 - 4\tilde{k}_n^2}, \qquad \text{if } m^2 > 4\tilde{k}_n^2. \end{aligned} \tag{3.18}$$



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The conformal dimension of the dual primary operator is just $h_R^s = -\Delta_s$.

Similar to the spacelike warped case, in order to compare with the prediction of dual CFT, one has to consider the identification of quantum numbersdue to the coordinates transformations. In the asymptotic region, we can make the following identification locally:

$$u^2 \leftrightarrow \frac{1}{r}, \qquad x^- \leftrightarrow \theta, \qquad x^+ \leftrightarrow -2t. \tag{3.19}$$

Correspondingly we have the identification between the quantum numbers:

$$k = -\tilde{\omega}_n, \tag{3.20}$$

$$\omega = 2\tilde{k}_n. \tag{3.21}$$

With this identification, the relation (3.10) can be rewritten as

$$\tilde{\omega}_R = -\tilde{k}_n - i2\pi T_R l(n + h_R) \tag{3.22}$$

This is reminiscent of the relation (1.1). The factor 2 discrepancy with (1.1) comes from the subtlety in defining the temperature. Actually, in some literatures, it is $2\pi$ rather than $4\pi$ appeared in (1.1).

Next, let us consider the quasi-normal modes of the massive vector field in the null warped black hole background. One can work with the following first order equations

$$\epsilon_\lambda^{\ \alpha\beta}\partial_\alpha A_\beta = -mA_\lambda. \tag{3.23}$$

On the null warped black hole background, they become

$$\frac{\mathrm{d}\phi_t}{\mathrm{d}r} = \left(-\frac{\omega k}{2mlr^2} - \frac{ml}{2r}\right)\phi_t - \frac{\omega^2}{2mlr^2}\phi_\theta, \tag{3.24}$$

$$\frac{\mathrm{d}\phi_\theta}{\mathrm{d}r} = \left(\frac{ml(r^2 + r + \alpha^2)}{2r^2} + \frac{k^2}{2mlr^2}\right)\phi_t + \left(\frac{ml}{2r} + \frac{\omega^2}{2mlr^2}\right)\phi_\theta, \tag{3.25}$$

$$\phi_r = \frac{-1}{2mlr^2}(ik\phi_t + i\omega\phi_\theta), \tag{3.26}$$

with the ansatz $A_\mu = e^{-i\omega t + ik\theta}\phi_\mu(r)$. From the above equations, we obtain a second order differential equation for $\phi_t$,

$$\frac{\mathrm{d}^2\phi_t}{\mathrm{d}x^2} + \frac{1}{4}\left(\frac{\omega^2 - m^2l^2 + 2ml}{x^2} + \frac{\omega^2 - 2\omega k}{x} + \omega^2\alpha^2\right)\phi_t = 0, \tag{3.27}$$

where $x = \frac{1}{r}$. It can be solved in terms of Kummer function analogous to the scalar field case

$$\phi_t = e^{-\frac{z}{2}} z^{\frac{1}{2} \pm \tilde{m}_v} F\left(\frac{1}{2} \pm \tilde{m}_v - \kappa, 1 \pm 2\tilde{m}_v, z\right), \tag{3.28}$$

where $z = -i\omega\alpha x$, $\tilde{m}_v = \frac{1}{2}\sqrt{(ml-1)^2 - \omega^2}$ and $\kappa = \frac{i}{4\alpha}(\omega - 2k)$. Note that $\phi_\theta, \phi_r$ can be solved straightforwardly by using (3.25) (3.26). Considering the boundary condition for the quasi-normal modes, we also have the similar relation as in the scalar case. One also has to make a combination of the two solutions with only ingoing modes at the horizon. The coefficients in the combination are

$$C_1 = -\frac{\Gamma(1 - 2\tilde{m}_v)}{\Gamma(\frac{1}{2} - \tilde{m}_v - \kappa)}C, \quad C_2 = \frac{\Gamma(1 + 2\tilde{m}_v)}{\Gamma(\frac{1}{2} + \tilde{m}_v - \kappa)}C, \tag{3.29}$$



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with $C$ being a constant. And the flux has a leading term proportional to $C_1^* C_2 - c.c.$ and the sub-leading term $C_2^* C_2 r^{-1+2\tilde{m}_v}$. So from the condition that the flux should be finite at asymptotic infinity, we have

$$\frac{1}{2} + \tilde{m}_v - \kappa = -n. \tag{3.30}$$

The conformal dimension of the massive vector field could be obtained easily:

$$h_R^v = \frac{1}{2} + \frac{1}{2}\sqrt{(ml-1)^2 - 4\tilde{k}_n^2}. \tag{3.31}$$

With this and the identification (3.20), we have the relation

$$\tilde{\omega}_R^v = -\tilde{k}_n - i2\pi T_R l(n + h_R^v), \tag{3.32}$$

from (3.30).

Lastly, we turn to the study of the quasi-normal modes of the fermionic perturbations. The analysis of the quasi-normal modes for the fermionic fields in the null warped black hole background is analogous to the stretched case. We first choose the vielbein:

$$\begin{aligned} e^0 &= rl\mathrm{d}t + \frac{1}{2}l(1-\alpha^2-r-r^2)\mathrm{d}\theta, \\ e^1 &= \frac{l}{2r}\mathrm{d}r, \\ e^2 &= -rl\mathrm{d}t + \frac{1}{2}l(1+\alpha^2+r+r^2)\mathrm{d}\theta \end{aligned}$$

and then calculate the spin connection. The none-zero components of the spin connection are

$$\begin{aligned} &\omega_0^{01} = -\omega_0^{10} = (1+\alpha^2-r^2)/l, \quad \omega_0^{12} = -\omega_0^{21} = (\alpha^2-r^2)/l, \quad \omega_1^{02} = -\omega_1^{20} = 1/l, \\ &\omega_2^{01} = -\omega_2^{10} = (\alpha^2-r^2)/l, \quad \omega_2^{12} = -\omega_2^{21} = (-1+\alpha^2-r^2)/l, \end{aligned}$$

where $\omega_a^{bc} = e_a^\mu \omega_\mu^{bc}$.

Taking the ansatz $\Psi_i = e^{-i\omega t + ik\theta}\psi_i(r)$ and making the redefinition of the fields as

$$\psi_1 = \psi_1' - \psi_2', \quad \psi_2 = \psi_1' + \psi_2', \tag{3.33}$$

we rewrite the Dirac equations as

$$\frac{\mathrm{d}\psi_1'}{\mathrm{d}r} = -\frac{2ml+3}{4r}\psi_1' + \frac{i\omega}{2r^2}\psi_2', \tag{3.34}$$

$$\frac{\mathrm{d}\psi_2'}{\mathrm{d}r} = \frac{i\omega(r^2+r+\alpha^2) - 2ikr}{2r^2}\psi_1' + \frac{2ml-1}{4r}\psi_2'. \tag{3.35}$$

As before we change the variable to $x = \frac{1}{r}$ and redefine the fields as $P = x^{-\frac{1}{2}}\psi_1'$, then we obtain a second order differential equation,

$$\frac{\mathrm{d}^2P}{\mathrm{d}x^2} + \frac{1}{4}\left\{\frac{1+\omega^2-(ml-\frac{1}{2})^2}{x^2} + \frac{\omega^2-2\omega k}{x} + \omega^2\alpha^2\right\}P = 0, \tag{3.36}$$



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which can be solved in terms of Kummer functions

$$\psi_1' = x^{\frac{1}{2}} e^{-\frac{z}{2}} z^{\frac{1}{2} \pm \tilde{m}_f} F\left(\frac{1}{2} \pm \tilde{m}_f - \kappa, 1 \pm 2\tilde{m}_f, z\right) \tag{3.37}$$

where $\tilde{m}_f = \frac{1}{2}\sqrt{(ml - \frac{1}{2})^2 - \omega^2}$, $\kappa = \frac{i}{4\alpha}(\omega - 2k)$.

The solution of $\psi_2'$ can be obtained by using (3.34). Similarly considering the boundary condition for the quasi-normal modes, we have

$$\frac{1}{2} + \tilde{m}_f - \kappa = -n. \tag{3.38}$$

The conformal dimension of the fermion operator is

$$h_R^f = \frac{1}{2} + \frac{1}{2}\sqrt{\left(ml - \frac{1}{2}\right)^2 - 4\tilde{k}_n^2}. \tag{3.39}$$

This relation and the identification (3.20) give us

$$\tilde{\omega}_R^f = -\tilde{k}_n - i2\pi T_R l(n + h_R^f), \tag{3.40}$$

from (3.38).

In short, the quasi-normal modes for various perturbations, including the massive scalar, vector and spin 1/2 fermion, of the null warped $AdS_3$ black hole could all be written in a concise form

$$\tilde{\omega}_R = -\tilde{k}_n - i2\pi T_R l(n + h_R). \tag{3.41}$$

This relation is quite similar to the prediction (1.1) of warped AdS/CFT correspondence, up to a factor 2. We take it as strong evidence to support the conjectured correspondence.

## 4 Conformal dimensions

In this section, we try to compute the conformal dimensions of the dual operators corresponding to various perturbations around the spacelike warped and null warped backgrounds. Instead of analyzing the asymptotic behavior of the solution of the equation of motions of the perturbations, we take a slightly more algebraic way. For both the spacelike stretched $AdS_3$ and the null warped $AdS_3$, they have the isometry group $SL(2, R) \times U(1)$. The perturbations should respect the isometry group. The highest conformal weight mode created by the bulk perturbations must obey the algebraic equation $L_1\phi = 0$. Its $L_0$ eigenvalue can be taken as the conformal dimensions of the primary operators in the boundary CFT. It turns out that the conformal dimensions determined in this way are in perfect agreement with the ones obtained before. For simplicity, we just focus on the scalar and the vector perturbations.

### 4.1 Spacelike stretched case

Let us consider a massive scalar $\Phi$ of mass $m$ in the warped spacelike $AdS_3$ spacetime. From warped AdS/CFT correspondence, such a scalar field have a counterpart boundary operator in dual conformal field theory. We work in the following form of the spacelike stretched $AdS_3$:

$$ds^2 = \frac{l^2}{\nu^2 + 3}\left(-\cosh^2\sigma d\tau^2 + d\sigma^2 + \frac{4\nu^2}{\nu^2 + 3}(du + \sinh\sigma d\tau)^2\right), \tag{4.1}$$



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which is the same as (2.22) by the coordinate transformation $r = \sinh\sigma, x = u$.

Such a background has the $U(1)_L \times SL(2,R)_R$ isometries. The $U(1)_L$ isometry is generated by

$$L_0 = i\partial_u, \tag{4.2}$$

and the $SU(2)_R$ isometry is generated by $\bar{L}_0, \bar{L}_1$ and $\bar{L}_{-1}$ satisfying

$$[\bar{L}_0, \bar{L}_{\pm 1}] = \mp \bar{L}_{\pm 1}, \qquad [\bar{L}_1, \bar{L}_{-1}] = 2\bar{L}_0, \tag{4.3}$$

where

$$\bar{L}_0 = i\partial_\tau, \tag{4.4}$$
$$\bar{L}_1 = -e^{i\tau}\left(\partial_\sigma + i\tanh\sigma\partial_\tau + i\frac{1}{\cosh\sigma}\partial_u\right), \tag{4.5}$$
$$\bar{L}_{-1} = e^{-i\tau}\left(\partial_\sigma - i\tanh\sigma\partial_\tau - i\frac{1}{\cosh\sigma}\partial_u\right). \tag{4.6}$$

The scalar equation of motion now takes the form

$$\begin{aligned}\frac{1}{\cosh\sigma}\partial_\sigma(\cosh\sigma\partial_\sigma)\Phi - \frac{1}{\cosh^2\sigma}\partial_\tau^2\Phi + \frac{2\sinh\sigma}{\cosh^2\sigma}\partial_\tau\partial_u\Phi \\ -\frac{\sinh^2\sigma}{\cosh^2\sigma}\partial_u^2\Phi + \left(\frac{\nu^2+3}{4\nu^2}\right)\partial_u^2\Phi - \frac{m^2l^2}{\nu^2+3}\Phi = 0.\end{aligned} \tag{4.7}$$

The above equation could be rewritten as

$$\left\{-\left[\frac{1}{2}(\bar{L}_1\bar{L}_{-1} + \bar{L}_{-1}\bar{L}_1) - \bar{L}_0^2\right] + \frac{3(\nu^2-1)}{4\nu^2}L_0^2 - \frac{m^2l^2}{\nu^2+3}\right\}\Phi = 0. \tag{4.8}$$

One may make the following ansatz

$$\Phi = e^{-i\tilde{\omega}_R\tau + i\tilde{k}u}\phi. \tag{4.9}$$

The corresponding highest weight mode should satisfy

$$\bar{L}_1\Phi = 0, \qquad \bar{L}_0\Phi = h_R\Phi. \tag{4.10}$$

This helps us to fix the conformal dimension[2]

$$h_R = \tilde{\omega}_R = \frac{1}{2} \pm \sqrt{\frac{1}{4} + s}, \tag{4.11}$$

where $s$ is defined to be (2.25). This is the same as (2.26), where we pick +.

For the highest weight state, we can even solve the equation (4.10) and get

$$\Phi = e^{-ih_R\tau + i\tilde{k}u}e^{\tilde{k}\tan^{-1}\sinh\sigma}(\cosh\sigma)^{-h_R}. \tag{4.12}$$

In order to study the conformal weight for the vector fields in the space-like warped $AdS_3$, it is more convenient to use the Poincare coordinates [9]

$$ds^2 = \frac{l^2}{\nu^2+3}\left(-x^2dt^2 + \frac{dx^2}{x^2} + \frac{4\nu^2}{\nu^2+3}(d\theta + xdt)^2\right). \tag{4.13}$$

[2]The conformal weights of the scalar fields have been discussed in [23] in the similar way independently. We would like to thank the anonymous referee for pointing this out.



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The Killing vectors of the space-like warped $\mathrm{AdS}_3$ are given by

$$V_{-1} = i\left(-\frac{1}{x^2} - t^2\right)\partial_t + 2itx\partial_x + \frac{2i}{x}\partial_\theta, \tag{4.14}$$

$$V_0 = t\partial_t - x\partial_x, \qquad V_1 = i\partial_t, \qquad V = i\partial_\theta, \tag{4.15}$$

which satisfy the commutation relations:

$$[V_0, V_{\pm 1}] = \mp V_{\pm 1}, \qquad [V_1, V_{-1}] = 2V_0. \tag{4.16}$$

For the highest weight state created by the vector field, it satisfies

$$\mathcal{L}_{V_1}A_\mu = 0, \qquad \mathcal{L}_{V_0}A_\mu = h_R^v A_\mu, \qquad \mathcal{L}_V A_\mu = -\tilde{k}A_\mu, \tag{4.17}$$

where $\mathcal{L}$ denotes Lie derivative. Then the solution is

$$A_t = C_1 x^{1-h_R^v} e^{i\tilde{k}\theta}, \qquad A_x = C_2 x^{-1-h_R^v} e^{i\tilde{k}\theta}, \qquad A_\theta = C_3 x^{-h_R^v} e^{i\tilde{k}\theta}. \tag{4.18}$$

Using the equation of motion for the vector fields, we obtain

$$C_2 = -\frac{i\tilde{k}(v^2+3)}{2mlv}C_1, \qquad C_3 = \frac{2mlv(h-1) - \tilde{k}^2(v^2+3)}{ml(2hv - ml)}C_1 \tag{4.19}$$

and

$$h_R^v = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{m^2l^2 - 2mlv}{v^2+3} - \frac{3(v^2-1)\tilde{k}^2}{4v^2}}. \tag{4.20}$$

This is slightly different from (2.56) by the sign before $m$, but is exactly the same as (2.60). The difference comes from that through the coordinate transformation, the helicity also exchange, which induce $m \to -m$.

It is interesting to compare the above results with the ones in usual $AdS_3/CFT_2$ correspondence. In latter case, one has

$$h_R + h_L = \Delta, \qquad h_R - h_L = \pm s, \tag{4.21}$$

where $s$ is the spin of the field,

$$\Delta = 1 \pm \sqrt{1 + m^2l^2}, \tag{4.22}$$

for the scalar fields, and

$$\Delta = 1 + |m|l, \tag{4.23}$$

for the vector fields. For warped $AdS_3/CFT_2$ correspondence, the relations (4.21) still make sense, even though we cannot determine $h_L$ directly. Another property shared by both cases is that in the vector and the fermionic case, when the helicity changes, $m$ changes sign and accordingly the expression of the conformal dimension need to be changed slightly. However, the relations (4.22), (4.23) have to be modified greatly in the warped AdS case. One modification is on the scale, from $l$ to $2l/\sqrt{v^2+3}$. Another modification is more significant: in the warped case, another quantum number from $\mathrm{U}(1)_R$ appears in the conformal dimensions. This is not only true for the spacelike stretched case but also true for the null warped case. As a consistent check, when $v = 1$, our result reduce to (4.22), (4.23).



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One interesting feature in the conformal weights of operators in dual CFT is that they depend on the U(1) quantum number $\tilde{k}$. The presence of this quantum number means that even though the mass-square of the scalar field satisfies the Breitenlohner-Freedman bound for three-dimensional AdS spacetime, the perturbation could still be unstable. As a result, superradiance may happen in the spacelike stretched $AdS_3$ spacetime [23], just like in Kerr black hole [26]. Similar phenomenon happens in the null warped $AdS_3$ spacetime as well.

### 4.2 Null warped case

For the null warped $AdS_3$ spacetime (3.13), it has isometry group $SL(2,R)_R \times U(1)_{null}$. The $U(1)_{null}$ is generated by

$$N = \partial_+, \tag{4.24}$$

and $SL(2,R)_R$ is generated by

$$\begin{aligned} N_1 &= \partial_-, \\ N_0 &= x^-\partial_- + \frac{u}{2}\partial_u, \\ N_{-1} &= (x^-)^2\partial_- - u^2\partial_+ + x^- u\partial_u, \end{aligned}$$

which satisfy the commutation relations:

$$[N_0, N_{\pm1}] = \mp N_{\pm1}, \qquad [N_1, N_{-1}] = 2N_0. \tag{4.25}$$

The equation of motion of the massive scalar $\Phi$ in the null warped $AdS_3$ spacetime is of the form

$$(u^2\partial_u^2 - u\partial_u - 4\partial_+^2 + 4u^2\partial_+\partial_- - m^2l^2)\Phi = 0, \tag{4.26}$$

which could be rewritten as

$$\left[\frac{1}{2}(N_1N_{-1} + N_{-1}N_1) - N_0^2 + N^2 + \frac{m^2l^2}{4}\right]\Phi = 0. \tag{4.27}$$

The highest conformal weight state should satisfy

$$N_1\Phi = 0, \qquad N_0\Phi = h_R^s\Phi, \qquad N\Phi = i\tilde{k}_n\Phi. \tag{4.28}$$

This just gives the constraint $\tilde{\omega} = 0$ and the conformal weight $h_R^s = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 + m^2l^2 - 4\tilde{k}_n^2}$. The solution of the highest weight state is

$$\Phi = u^{2h_R} e^{i\tilde{k}_n x^+} \tag{4.29}$$

The consistent boundary condition require $h_R^s > 0$.

For the conformal weight of the vector field, the highest weight state satisfy

$$\mathcal{L}_{N_1}A_\mu = 0, \qquad \mathcal{L}_{N_0}A_\mu = h_R^v A_\mu, \qquad \mathcal{L}_N A_\mu = i\tilde{k}_n A_\nu. \tag{4.30}$$

This is consistent with the equation of motion. For any Killing vector $\xi$, we can choose a coordinate $y$ satisfying $\xi = \partial_y$. In this special coordinates, the background metric is independent of $y$. The



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equation of motion for the vector fields can be seen as a linear operator acting on the vector fields. Since the operator only depends on the metric, the operator commutes with the Lie derivative of the Killing vector. So the equations for the highest weight solution are consistent with the equations of the vector fields. The solution of (4.30) is

$$A_+ = C_1 u^{2h_R^v} e^{i\tilde{k}_n x^+}, \qquad A_- = C_3 u^{2h_R^v} e^{i\tilde{k}_n x^+}, \qquad A_u = C_2 u^{2h_R^v - 1} e^{i\tilde{k}_n x^+}. \tag{4.31}$$

Using the equation of motion for the vector fields, we obtain

$$C_1 = \frac{-2\tilde{k}_n^2}{ml(h_R^v - ml)} C_2, \qquad C_3 = \frac{-2i\tilde{k}_n}{ml} C_2, \tag{4.32}$$

and also find the conformal dimension

$$h_R^v = \frac{1}{2} + \frac{1}{2}\sqrt{(ml-1)^2 - 4\tilde{k}_n^2}. \tag{4.33}$$

Comparing with the conformal dimensions in usual AdS/CFT correspondence, we see that the only difference is the appearance of $\tilde{k}$ terms in (4.33).

## 5 Conclusions and discussion

In this paper, we calculated the quasi-normal modes of various perturbations, including the massive scalar, vector and spin one-half fermionic perturbations, of the spacelike stretched and the null warped $AdS_3$ black holes. For the spacelike stretched black hole, all kinds of the quasi-normal modes could be rewritten in terms of the quantum numbers $\tilde{\omega}$ and $\tilde{k}$ in a simple way:

$$\begin{aligned} \tilde{\omega}_R &= \frac{1}{\nu^2 + 3}(-4\pi T_L l\tilde{k} - (i4\pi T_R l)(n_1 + h_R)), \qquad \text{or} \\ \tilde{k} &= -i(n_2 + h_L), \end{aligned} \tag{5.1}$$

where $n_1, n_2$ are non-negative integers. Similarly, for the null warped black hole, the quasi-normal modes are of the form

$$\tilde{\omega}_R = -\tilde{k}_n - i2\pi T_R l(n + h_R), \tag{5.2}$$

with $n$ being non-negative integer. The above relations are reminiscent of the relations (1.1) on the poles of the retarded Green's function. Since the conjectured correspondence is between the spacelike stretched (null) warped $AdS_3$ and its holographically dual 2D CFT, one needs to use the quantum numbers appeared in these spacetimes rather than the ones in the black holes to set up the dictionary. This is why we rewrite the quasi-normal modes in terms of the quantum numbers $\tilde{\omega}$ and $\tilde{k}$ of the spacelike (null) warped $AdS_3$. And actually it is in terms of these quantum numbers that make the warped AdS/CFT correspondence manifest. This is the key point to set up the dictionary. The phenomena happening here is extraordinary. The asymptotic geometries of the warped black holes could be locally transformed to the ones of the warped spacetimes. The coordinate transformations induce the identifications of two sets of quantum numbers. Taken this subtlety into account, the quasi-normal modes of the warped black holes could be reorganized into (5.1), (5.2), which are well consistent with the CFT prediction (1.1) and so support the conjectures.



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The relations (5.1), (5.2) were obtained after the identifications (2.29), (3.20) being taken into account. However, the identifications come from the local transformations, rather than the global ones. Especially, considered the fact that the global warped $AdS_3$ spacetime is not included in the black hole's phase space, the above identifications deserve further investigations and clarifications. We wish we can return to this issue in the future.

One interesting point is that it seems that we have only one set of the quasi-normal modes. For the null warped case, this seems to be natural since the dual CFT has only right temperature. For the spacelike stretched case, this sounds strange. However, recall that the isometry group of the spacelike stretched $AdS_3$ is just $SL(2,R)_R \times U(1)_L$. For the highest weight operators corresponding to the massive scalar, one can define its right-moving conformal weight from $SL(2,R)_R$, but can only define $\tilde{k}$ from $U(1)_L$. This in fact is in consistence with (5.1).

Another interesting point in our result is that the above two relations (5.1), (5.2) are still a little different from the predicted poles (1.1), up to a scale factor. This could be due to the ambiguity in determining the temperature, originated from coordinate transformation. This possibility has been shown in [10] for comparison with the BTZ black hole. It would be interesting to pin down this issue.

In this paper, we proposed a conjecture that the quantum topological massive gravity asymptotic to the null warped $AdS_3$ is holographically dual to 2D CFT. Our study on the quasi-normal modes of the null warped black hole support this conjecture. However it would be essential to put this conjecture on a more solid ground. One interesting issue is that the holographic anomaly suggests that there should be not only right sector but also left one as well. This could not be seen from the study of the black hole thermodynamics and the quasi-normal modes. To understand this issue better, it would be important to investigate the asymptotical boundary conditions on the gravitational perturbations and check if the central charges could be derived from the symmetry algebra.

It would be worth looking for other evidence to support the warped AdS/CFT correspondence. One possibility is to compare the absorption cross sections. This has been explored in the context of Kerr/CFT correspondence [26]. For the warped $AdS_3$ black hole, since the equation of motion of the perturbations are exactly solvable, we expect that the same analysis would be feasible [27].

In this paper, we discussed the quasi-normal modes of the scalar, vector and fermionic perturbations. It would be interesting to consider the gravitational perturbations. In this case, the equations of motions is a third order differential equation, so is more difficult to solve. Nevertheless, It was shown in [19] that after fixing the gauge completely, the equations of motion in the warped $AdS_3$ could be simplified. We expect the same simplification may happen in the warped black hole case.


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## References


[1] H.P. Nollert, *Quasinormal modes: the characteristic 'sound' of black holes and neutron stars*, *Class. Quant. Grav.* **16** (1999) R159 [SPIRES];
K.D. Kokkotas and B.G. Schmidt, *Quasi-normal modes of stars and black holes*, *Living Rev. Rel.* **2** (1999) 2 [gr-qc/9909058] [SPIRES];
E. Berti, V. Cardoso and A.O. Starinets, *Quasinormal modes of black holes and black branes*, *Class. Quant. Grav.* **26** (2009) 163001 [arXiv:0905.2975] [SPIRES].

[2] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [hep-th/9711200] [SPIRES];
S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109] [SPIRES];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150] [SPIRES].

[3] E. Witten, *Anti-de Sitter space, thermal phase transition and confinement in gauge theories*, *Adv. Theor. Math. Phys.* **2** (1998) 505 [hep-th/9803131] [SPIRES].

[4] G.T. Horowitz and V.E. Hubeny, *Quasinormal modes of AdS black holes and the approach to thermal equilibrium*, *Phys. Rev.* **D 62** (2000) 024027 [hep-th/9909056] [SPIRES].

[5] B. Wang, C.-Y. Lin and E. Abdalla, *Quasinormal modes of Reissner-Nordstróm anti-de Sitter black holes*, *Phys. Lett.* **B 481** (2000) 79 [hep-th/0003295] [SPIRES].

[6] D. Birmingham, *Choptuik scaling and quasinormal modes in the AdS/CFT correspondence*, *Phys. Rev.* **D 64** (2001) 064024 [hep-th/0101194] [SPIRES];
D. Birmingham, I. Sachs and S.N. Solodukhin, *Conformal field theory interpretation of black hole quasi- normal modes*, *Phys. Rev. Lett.* **88** (2002) 151301 [hep-th/0112055] [SPIRES].

[7] D.T. Son and A.O. Starinets, *Minkowski-space correlators in AdS/CFT correspondence: Recipe and applications*, *JHEP* **09** (2002) 042 [hep-th/0205051] [SPIRES].

[8] M. Bañados, C. Teitelboim and J. Zanelli, *The black hole in three-dimensional space-time*, *Phys. Rev. Lett.* **69** (1992) 1849 [hep-th/9204099] [SPIRES].

[9] D. Anninos, W. Li, M. Padi, W. Song and A. Strominger, *Warped* $AdS_3$ *black holes*, *JHEP* **03** (2009) 130 [arXiv:0807.3040] [SPIRES].

[10] B. Chen and Z.-b. Xu, *Quasinormal modes of warped* $AdS_3$ *black holes and AdS/CFT correspondence*, arXiv:0901.3588 [SPIRES].

[11] S. Deser, R. Jackiw and S. Templeton, *Topologically massive gauge theories*, *Ann. Phys.* **140** (1982) 372 [*Erratum ibid.* **185** (1988) 406] [*Annals Phys.* **281** (2000) 409] [SPIRES].

[12] S. Deser, R. Jackiw and S. Templeton, *Three-dimensional massive gauge theories*, *Phys. Rev. Lett.* **48** (1982) 975 [SPIRES].

[13] G. Compere and S. Detournay, *Semi-classical central charge in topologically massive gravity*, *Class. Quant. Grav.* **26** (2009) 012001 [*Erratum ibid.* **26** (2009) 139801] [arXiv:0808.1911] [SPIRES].

[14] Y. Nutku, *Exact solutions of topologically massive gravity with a cosmological constant*, *Class. Quant. Grav.* **10** (1993) 2657 [SPIRES].

[15] M. Gurses, *Perfect fluid sources in* $2+1$ *dimensions*, *Class. Quant. Grav.* **11** (1994) 2585.

[16] K.A. Moussa, G. Clement and C. Leygnac, *The black holes of topologically massive gravity*, *Class. Quant. Grav.* **20** (2003) L277 [gr-qc/0303042] [SPIRES].




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[17] A. Bouchareb and G. Clement, *Black hole mass and angular momentum in topologically massive gravity*, *Class. Quant. Grav.* **24** (2007) 5581 [arXiv:0706.0263] [SPIRES].

[18] E. Ayon-Beato and M. Hassaine, *pp waves of conformal gravity with self-interacting source*, *Ann. Phys.* **317** (2005) 175 [hep-th/0409150] [SPIRES]; *Exploring AdS waves via nonminimal coupling*, *Phys. Rev.* **D 73** (2006) 104001 [hep-th/0512074] [SPIRES].

[19] D. Anninos, M. Esole and M. Guica, *Stability of warped AdS3 vacua of topologically massive gravity*, *JHEP* **10** (2009) 083 [arXiv:0905.2612] [SPIRES].

[20] M. Blagojevic and B. Cvetkovic, *Asymptotic structure of topologically massive gravity in spacelike stretched AdS sector*, *JHEP* **09** (2009) 006 [arXiv:0907.0950] [SPIRES].

[21] G. Compere and S. Detournay, *Boundary conditions for spacelike and timelike warped $AdS_3$ spaces in topologically massive gravity*, *JHEP* **08** (2009) 092 [arXiv:0906.1243] [SPIRES].

[22] G. Compere, S. Detournay and M. Romo, *Supersymmetric Gödel and warped black holes in string theory*, *Phys. Rev.* **D 78** (2008) 104030 [arXiv:0808.1912] [SPIRES];
D. Anninos, *Hopfing and puffing warped Anti-de Sitter space*, *JHEP* **09** (2009) 075 [arXiv:0809.2433] [SPIRES];
J.J. Oh and W. Kim, *Absorption cross section in warped $AdS_3$ black hole*, *JHEP* **01** (2009) 067 [arXiv:0811.2632] [SPIRES];
W. Kim and E.J. Son, *Thermodynamics of warped $AdS_3$ black hole in the brick wall method*, *Phys. Lett.* **B 673** (2009) 90 [arXiv:0812.0876] [SPIRES];
S. Nam and J.-D. Park, *Hawking radiation from covariant anomalies in* $2+1$ *dimensional black holes*, *Class. Quant. Grav.* **26** (2009) 145015 [arXiv:0902.0982] [SPIRES];
Y.S. Myung, *Entropy of extremal warped black holes*, *Mod. Phys. Lett.* **A 24** (2009) 1485 [arXiv:0902.1566] [SPIRES];
S.K. Chakrabarti, P.R. Giri and K.S. Gupta, *Scalar field dynamics in warped $AdS_3$ black hole background*, *Phys. Lett.* **B 680** (2009) 500 [arXiv:0903.1537] [SPIRES].

[23] D. Anninos, *Sailing from warped $AdS_3$ to warped $dS_3$ in topologically massive gravity*, arXiv:0906.1819 [SPIRES].

[24] M. Guica, T. Hartman, W. Song and A. Strominger, *The Kerr/CFT correspondence*, arXiv:0809.4266 [SPIRES].

[25] W. Li, W. Song and A. Strominger, *Chiral gravity in three dimensions*, *JHEP* **04** (2008) 082 [arXiv:0801.4566] [SPIRES];
A. Strominger, *A simple proof of the chiral gravity conjecture*, arXiv:0808.0506 [SPIRES];
A. Maloney, W. Song and A. Strominger, *Chiral gravity, log gravity and extremal CFT*, arXiv:0903.4573 [SPIRES].

[26] I. Bredberg, T. Hartman, W. Song and A. Strominger, *Black hole superradiance from Kerr/CFT*, arXiv:0907.3477 [SPIRES].

[27] B.Chen, B. Ning and Z.B. Xu, work in progress.


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